

(For the exercise sessions on 19 February 2026.)

Exercise P1.1 – Connectivity

Let $G = (V, E)$ be a two connected graph.

- (a) Let (u, v, w) be a path in G of length 2. Show that we can extend this path to a cycle, that is, show that G contains a cycle where u, v, w appear as incident vertices.
- (b) Let $e \in E$ be an edge, and $u, v \in V$ be two vertices. Show that there is a path in G from u to v passing through e .
(*Hint: you may use without proof that in a 2-connected graph, every two edges e, f lie on a common cycle.*)

Solution 1

Connectivity

- (a) Let (u, v, w) be a path in G of length 2. By the 2-connectedness of G , the graph $G[V \setminus v]$ is connected. Hence, there is a u - w path P that does not contain v . Combining P and (u, v, w) gives the desired cycle.
- (b) Let $e \in E$ be an edge, and $u, v \in V$ be two vertices. Consider the graph $G' = (V, E \cup \{\{u, v\}\})$ that arises from G by adding the edge $\{u, v\}$ (if it did not exist already anyways). The graph G' is also 2-connected. Applying the hint to the edges e and $\{u, v\}$, we obtain a cycle C containing both e and $\{u, v\}$. Removing $\{u, v\}$ from this cycle gives the desired path.