

Graph colouring extra exercise

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Exercise 1 – *Graph coloring*

- (a) Let $G = (V, E)$ be a graph, and $w \in V$ be a vertex, such that for all $v \in V \setminus \{w\}$ we have $\deg(v) \leq k - 1$. Show that G has a proper coloring into at most k colors.
- (b) Let $G = (V, E)$ be a graph, and let $S \subset V$ be a set of all vertices with degree larger than $k - 1$. Show that if $|S| \leq k$, there is a proper coloring of G into at most k colors. Describe an algorithm finding such a coloring in time $O(|V| + |E|)$.
- (c) Let $G = (V, E)$ be a graph with $|E| < \binom{k}{2}$. Show that there exist a proper coloring of G into k colors.