

Graph colouring extra exercise

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Exercise 1 – Graph coloring

(a) Let $G = (V, E)$ be a graph, and $w \in V$ be a vertex, such that for all $v \in V \setminus \{w\}$ we have $\deg(v) \leq k - 1$. Show that G has a proper coloring into at most k colors.

1. Choose an order for Greedy alg.
2. Show Greedy never assigns a colour $> k$

GREEDY-FÄRBUNG (G)

- 1: wähle eine beliebige Reihenfolge der Knoten: $V = \{v_1, \dots, v_n\}$
- 2: $c[v_1] \leftarrow 1$
- 3: for $i = 2$ to n do
- 4: $c[v_i] \leftarrow \min\{k \in \mathbb{N} \mid k \neq c(u) \text{ für alle } u \in N(v_i) \cap \{v_1, \dots, v_{i-1}\}\}$

(1) v_1, v_2, \dots, v_n s.t. $v_1 = w$

(2) for all $j > 1$ we never assign a colour $> k$, because:

$$|N(v_j) \cap \{v_1, \dots, v_{j-1}\}| \leq |N(v_j)| \leq k-1 \quad (\deg(v_j) \leq k-1, \text{ for } j > 1)$$

for $j=1$ we never assign a colour $> k$, because:

$$|N(v_1) \cap \{\emptyset\}| = 0 \leq k-1$$

- (b) Let $G = (V, E)$ be a graph, and let $S \subset V$ be a set of all vertices with degree larger than $k - 1$. Show that if $|S| \leq k$, there is a proper coloring of G into at most k colors. Describe an algorithm finding such a coloring in time $O(|V| + |E|)$.
- (c) Let $G = (V, E)$ be a graph with $|E| < \binom{k}{2}$. Show that there exist a proper coloring of G into k colors.