

Exercise S4.1 – Perfect Bipartite Matching

Let $G = (A \cup B, E)$ be a bipartite k -regular graph (i.e. every vertex has degree exactly k).

- (a) Show that G contains a perfect matching.
 (b) Show that G contains k pairwise disjoint perfect matchings.
Hint: use (a)

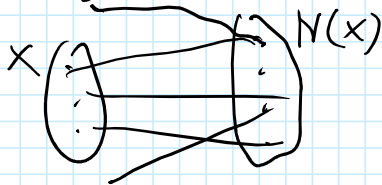
a) G is bipartite, G is k -regular
 $\Rightarrow k \cdot |A| = |E| = k \cdot |B|$
 $\Rightarrow |A| = |B|$

Theorem: (Hall, 1935)
 Ein bipartiter graph $G=(A \cup B, E)$ enthält ein
 Matching M der Kardinalität $|M|=|A|$ gdw
 $\forall A' \subseteq A : |A'| \leq |N(A')|$

Let $X \subseteq A$

wts $|X| \leq |N(X)|$

$k \cdot |X|$ edges are incident to X and all of them end in $N(X)$



There are $k \cdot |N(X)|$ edges incident to $N(X)$, among them are all $k|X|$ edges originating in X

$$\Rightarrow k \cdot |X| \leq k \cdot |N(X)|$$

$$\Rightarrow |X| \leq |N(X)|$$

$\Rightarrow \exists$ matching M with $|M|=|A|$ (Hall)

$\Rightarrow M$ is perfect ($|A|=|B|$)