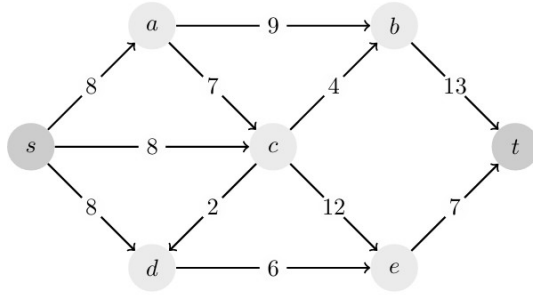


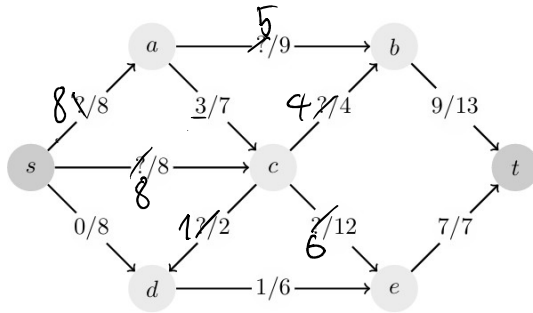
S11

Exercise S11.1 – Flows

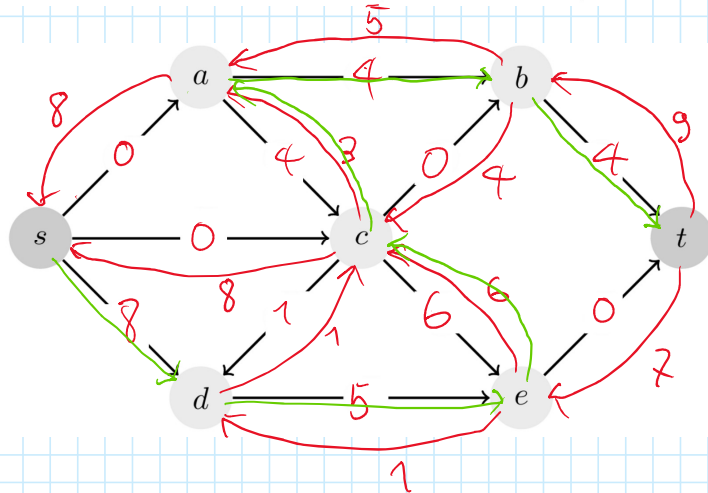
Consider the following network N . The number at each edge represents its capacity.



(a) Fill in the missing values so that they form a feasible (not necessarily maximum) flow:

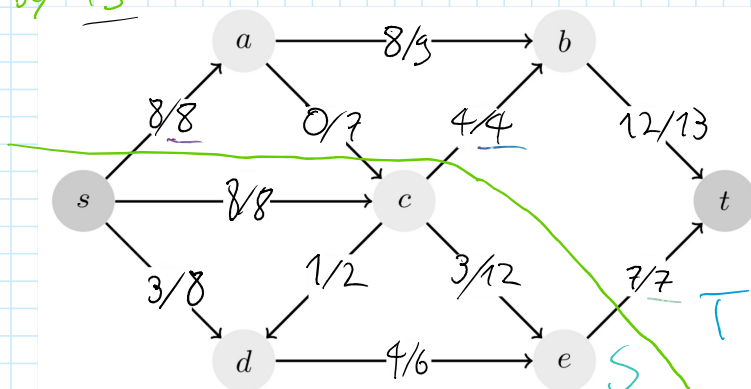


(b) Construct the residual network (“Restnetzwerk”).

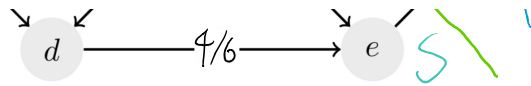


(c) Find an augmenting s - t -path and augment the flow along this path. If necessary, repeat this step until you have found a maximum flow.

We can increase along $s \rightarrow c \rightarrow b \rightarrow t$ to increase the flow by +3

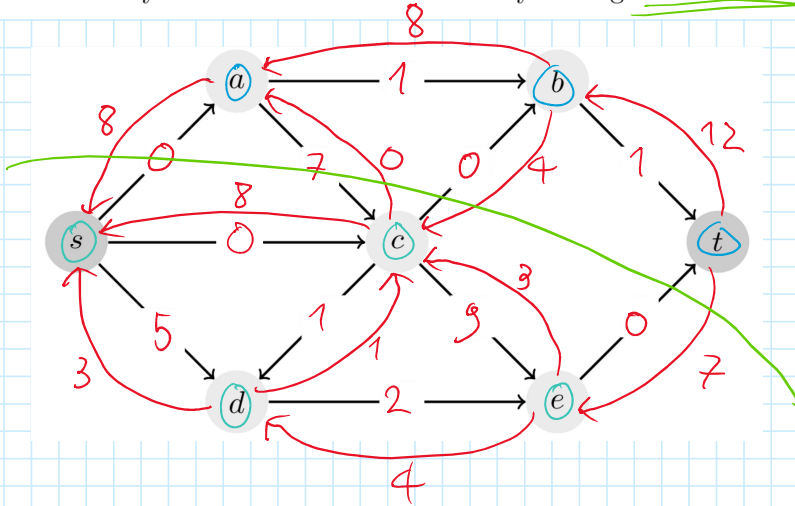


new $\text{val}(f) = 15$



(d) Prove that your flow is a maximum flow by finding a minimum cut in N .

$$\underline{\text{val}(f)} = 19 = \text{Cap}(S, T)$$



$$S := \{v \in V \mid v \text{ reachable from } s \text{ in } N_f\}$$

$$T := V/S$$

See: For (S, T) we have $f(S, T) = \text{Cap}(S, T) = 19$

$\text{val}(f)$ is $\max\{\text{flow}\}$, and $\text{Cap}(S, T)$ is $\min\{\text{cut}\}$

(See Thm. 3.11)