

# Notes S3

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## Exercise S3.1 – *Bipartite Matching*

Let  $G = (A \cup B, E)$  be a bipartite graph.

(a) Prove or disprove: If  $G$  has a Hamiltonian cycle, then  $G$  contains two disjoint perfect matchings.

(b) Prove or disprove: If  $G$  contains two disjoint perfect matchings, then  $G$  has a Hamiltonian cycle.

- (c) Let  $A' \subseteq A$  and  $B' \subseteq B$ . Assume that there is a matching  $M_A$  covering  $A'$  and a matching  $M_B$  covering  $B'$  (these two matchings do not have to be disjoint!). Show that there is a matching  $M$  that covers both  $A'$  and  $B'$  (i.e. it covers  $A' \cup B'$ ). (*Hint:* Which properties has the graph  $(V, M_A \cup M_B)$ ? Try to build a matching using only edges in  $M_A \cup M_B$ .)