S5

18 March 2026 18:35

Exercise S5.1 – *3-colorable graphs*

In the lecture you have seen an algorithm that finds an $O\left(\sqrt{|V|}\right)$-coloring for every 3-colorable graph (V, E). Fix $\alpha = \sqrt{|V|}$. The algorithm goes as follows.

1. While there is a vertex v with degree at least α, color v with a new color, color the neighbors of v with two additional new colors, and delete v and all its neighbors from the graph.
2. Color all remaining vertices with at most $\alpha + 1$ colors.

We want to analyze and generalize this algorithm.

(a) Where in the algorithm do we use that the graph is 3-colorable?

(b) How many colors do we need at most (depending on α)? Show that for $\alpha = \sqrt{|V|}$ we only need $O\left(\sqrt{|V|}\right)$ colors.

(c) Show that your bound in (a) is tight. More precisely, create a 3-colorable graph on which the algorithm uses $\Omega\left(\sqrt{|V|}\right)$ colors.

(d) Can you describe an algorithm that finds a $O\left(|V|^{\frac{2}{3}}\right)$-coloring for every 4 colorable graph?

(e) Can you describe an algorithm that finds a $O\left(|V|^{\frac{q-2}{q-1}}\right)$-coloring for every q colorable graph, where $q \geq 2$ is a constant?