S6

26 March 2026 13:20

## Exercise S6.1 – *Probability Space*

(a) For each of the following subtasks, either define a probability space and events $A$ and $B$ (and $C$) with the described properties, or prove that such a space cannot exist. Make sure that you define both, the sample space ("Ergebnismenge") $\Omega$ and the probabilities of the atomic events ("Elementarereignisse").

(i) $\Pr[A] = \frac{1}{4}$, $\Pr[B] = \frac{1}{3}$ and $\Pr[A \cup B] = \Pr[A] + \Pr[B]$.

(ii) $\Pr[A] = \frac{1}{4}$, $\Pr[B] = \frac{1}{3}$ and $\Pr[A \cup B] < \Pr[A] + \Pr[B]$.

(iii) $\Pr[A] = \Pr[B]$, $\Pr[A \cap B] = \frac{1}{4}$, and $\Pr[A \cap B] = \Pr[A] \cdot \Pr[B]$ (that is $A$ and $B$ are independent).

(iv) $\Pr[A] = \Pr[B] = \Pr[C] = \frac{5}{6}$ and $\Pr[A \cap B \cap C] = 0$.

(b) Samantha has a fair, six-sided die and a 5 CHF coin. She rolls the die and tosses the coin. Samantha considers her experiment a success if the coin shows a strictly larger value than the die (for the coin, heads is counted as 0; tails is counted as 5). Model her experiment with a suitable probability space. Explicitly define the event $A$ that the experiment is a success and determine its probability $\Pr[A]$.

(c) Oliver owns three pairs of shoes – two blue pairs, and one yellow, which he stores unordered in his wardrobe. One morning, during a power outage, he has to put on his shoes in complete darkness. He randomly (uniformly at random) grabs two shoes from the wardrobe and tries to put them on.

We let $A$ denote the event that he picked one left shoe and one right shoe (i.e. he is able to put on the shoes he picked), and we let $B$ be the event that the two shoes he picked have the same color.

Model this setting as a probability space and compute $\Pr[A]$ and $\Pr[A|B]$.