S8

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Exercise S8.1 – *Inequalities*

We throw 1000 fair coins and denote the results by $C_1, \ldots, C_{1000}$. We want to count the number of neighbouring coins that both show "heads". Two neighbouring coins are coins of the form C_i, C_{i+1} or C_{1000}, C_1 (imagine the coins being placed on a circle). Let X denote the number of neighbouring coins that both show "heads". We will use different inequalities to bound the probability that X is significantly larger than its expected value. Make sure to check all relevant conditions before applying an inequality.

(a) Show that $\mathbb{E}[X] = 250$.

(b) Use Markov's inequality to bound $\Pr[X \geq 300]$.

(c) Compute $\mathrm{Var}[X]$ and use Chebychev's inequality to bound $\Pr[X \geq 300]$.

(d) We define Y as the number of neighbouring coins that both show "heads" and for which the first coin has an odd index. That is, we only consider pairs of the form X_{2i-1}, X_{2i}, with $i = 1, \ldots, 500$. Show that $\mathbb{E}[Y] = 125$ and use Chernoff's bound for $\Pr[Y \geq 150]$.

(e) Use (d) to bound $\Pr[X \geq 300]$.
Remark: Could we just apply Chernoff's bound right away?