S12

Exercise S12.1 – *Integrality and Matching*

We have already seen Frobenius' Theorem: *Let $k \geq 1$. Every k-regular bipartite graph contains a perfect matching.* The proof we have seen uses Hall's theorem. In this exercise, we want to find another proof using our knowledge about flows.

Let $G = (A \dot{\cup} B, E)$ be a k-regular graph for $k \geq 1$.

(a) Describe how to model bipartite matching on G as a flow problem in a network $N = (V, A, c, s, t)$.

(b) Construct explicitly a (not necessarily integer!) flow f on N with $\text{val}(f) = n$. Conclude that $\text{maxflow}(N) = n$.
Hint: you don't have to construct an integral flow. Make sure to define the flow value on all edges of your network N!

(c) Using the result from (b), show that there exists an integer valued flow f' with $\text{val}(f') = n$. Using f', prove that G contains a perfect matching M.