A&W 2026

G-13 Timo Stucki, LFW E 13

Week 0

Semester and Exam Overview

Warm-Up Exercises

Infos

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Material:

- timostucki.com

About this exercise group

- Listed as focus group
  - Focus on helping you pass the exam (and get a good grade!)
  - Teaching will be centred around preparing you for the exam → less time for in-depth understanding of theory and hard proofs, more time for exercise walk-throughs and what's most important for the exam
  - I'm there for you if you are struggling
    - Never hesitate to ask questions!
    - Contact me
- What I will not do
  - Skip material
  - Go at a slower pace in the sense that we fall behind with material

(In my opinion, a focus group should neither give you false comfort nor put you at a disadvantage)

Semester and Exam Overview

Exam

- Points:
  - Moodle: 50%
  - CodeX: 25%
  - Written exercises (proofs): 25%
- Time: 3h
- Structure:
  - Moodle: 4 parts, mostly similar to semester mini quizes, some are more similar to short exercises from old exams
  - CodeX: 2 parts, like semester CodeX, usually 1 DP and 1 flow/graph
  - Written exercises: 2 parts with subtasks, similar to proofs from the lecture / in the script
  - All parts are equal in points

During the semester

- Mini quiz: Every two weeks
  - At the beginning of the exercise session
  - 2 Points
  - Good prep for Moodle exam
- CodeX: Every week
  - 2 Points
  - Good prep for CodeX exam
- Written exercises: Every week
  - Switches between regular theory exercises and peer grading exercises
  - 2 Points
  - Good prep for written exam (last year peer grading exercises were different from exam exercises)

Bonus:

80% of semester points give 0.25 grade bonus

Exam prep after the semester

- Moodle
  - Mini quizes (from current and prev. semesters)
  - Short exercises from old written exams
  - (Anki deck)
- CodeX
  - Semester CodeX tasks
  - Prev. CodeX exams
- Written exercises
  - Understanding proofs and learning them by heart / reading script
  - Semester written exercises
  - Long exercises from old written exams (proofs)

Warm-Up Exercises

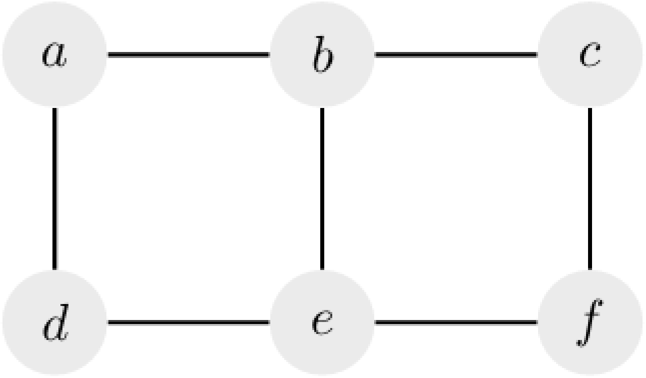
A&D Recap

	"non-distinct" vertices	distinct vertices
-	**walk**	**path**
endpoints must be the same	**closed walk** (German: Zyklus)	**cycle** (German: Kreis)

$O(1) \subset O(\log\log n) \subset O(\log n) \subset O(\sqrt{n}) \subset O(n) \subset O(n\log n) \subset O(n\sqrt{n}) \subset O(n^2) \subset O(2^n) \subset O(n!) \subset O(n^n)$

Exercise 0.1 – *Paths, Walks, Cycles*

Consider the following graph $G = (V, E)$.



1. Which paths of length 4 (i.e. consisting of 4 edges) are there from a to e?
2. Which walks of length 4 (i.e. consisting of 4 edges) are there from a to e?
3. Which cycles are there in G?
4. How many closed walks are there in G?

A&D Recap

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endpoints must be the same	**closed walk** (German: Zyklus)	**cycle** (German: Kreis)

$$O(1) \subset O(\log\log n) \subset O(\log n) \subset O(\sqrt{n}) \subset O(n) \subset O(n\log n) \subset O(n\sqrt{n}) \subset O(n^2) \subset O(2^n) \subset O(n!) \subset O(n^n)$$

Exercise 0.2 – *Asymptotic Growth*

(a) (Simpler) Sort the following functions asymptotically, i.e., according to O-notation. Here, $\log n$ denotes the logarithm to base 2 of n, and $\ln n$ denotes the natural logarithm of n. In which cases do you have asymptotic equality $\Theta(\cdot)$?

$$n, \quad 0.01n^2, \quad e^n, \quad \log n, \quad 2^{32}, \quad 2^n, \quad n + \sqrt{n},$$

(b) (Harder) Additionally, insert the following functions into your sequence.

$$\ln n, \quad \frac{n}{\log n}, \quad e^{\sqrt{\log n}}, \quad \log(n^2), \quad n^{1/4}, \quad n!$$

Exercise 0.3 – *Induction*

(a) Show that for all $n \in \mathbb{N}$:

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\ldots+\frac{1}{n\cdot(n+1)}=\frac{n}{n+1}.$$

(b) Prove *Bernoulli's inequality*: For all natural numbers $n \geq 1$ and all $h \in \mathbb{R}$ with $h \geq -1$, the following holds:

$$1+nh \leq (1+h)^n.$$

Exercise 0.4 – *A General Property of Graphs*

Prove that every graph G with $n \geq 2$ vertices contains two distinct vertices $v \neq w$ such that $\deg(v) = \deg(w)$.

Hint: For a given n, what is the maximum possible degree a vertex can have?

Exercise 0.5 – *Algorithm*

Describe an algorithm that solves the following problem: Given an input consisting of a graph $G = (V, E)$ with n vertices (assume the graph is given as an adjacency list), your algorithm should output "Yes" if G is a tree and "No" otherwise.

As always, a complete solution should include: a clear description of the algorithm, a proof of correctness, and a runtime analysis.

Hint: For this exercise, you may use the statement from Exercise 6 without proof.

Exercise P1.1

Connectivity

Lecture Recap

Definition: Sei $G = (V, E)$ ein Graph.

G heisst k-zusammenhängend, wenn gilt:

- $|V| \geq k + 1$ und
- $\forall X \subseteq V$ mit $|X| < k$ ist $G[V \setminus X]$ zusammenhängend

Satz von Menger:

Sei $G = (V, E)$ ein Graph. Dann gilt:

G ist **k-zusammenhängend** genau dann wenn (gdw.)

$\forall u, v \in V, u \neq v$ gibt es **k intern-knotendisjunkte** u-v-Pfade

Lecture Recap

Definition: Sei $G = (V, E)$ ein Graph.

G heisst k-kanten-zusammenhängend, wenn gilt:

- $\forall X \subseteq E$ mit $|X| < k$ ist $(V, E \setminus X)$ zusammenhängend

Satz von Menger (Kanten-Version):

Sei $G = (V, E)$ ein Graph. Dann gilt:

G ist **k-kanten-zusammenhängend** genau dann wenn (gdw.)

$\forall u, v \in V, u \neq v$ gibt es **k kantendisjunkte** u-v-Pfade

Exercise P1.1 – *Connectivity*

Let $G = (V, E)$ be a two connected graph.

(a) Let (u, v, w) be a path in G of length 2. Show that we can extend this path to a cycle, that is, show that G contains a cycle where u, v, w appear as incident vertices.

(b) Let $e \in E$ be an edge, and $u, v \in V$ be two vertices. Show that there is a path in G from u to v passing through e.
(*Hint:* you may use without proof that in a 2-connected graph, every two edges e, f lie on a common cycle.)

!!! WICHTIG !!!

Es gilt immer:

(Knoten-)Zusammenhang ≦ Kanten-Zusammenhang ≦ minimaler Grad